

2003000204040057
EXAMINATION SEPTEMBER 2024 (ATKT EXAM)
BACHELOR OF SCIENCE (MATHEMATICS) (FOURTH SEMESTER)
E.G-4001: MATHEMATICAL MODELING

[Time: As Per Schedule]

[Max. Marks:50]

Instructions:

1. Fill up strictly the following details on your answer book

- a. Name of the Examination: **BACHELOR OF SCIENCE (MATHEMATICS) (FOURTH SEMESTER)**
 - b. Name of the Subject: **E.G-4001: MATHEMATICAL MODELING**
 - c. Subject Code No: **2003000204040057**
2. Sketch neat and labelled diagram wherever necessary.
 3. Figures to the right indicate full marks of the question.
 4. All questions are compulsory.
 5. Follow usual notations.
 6. Use scientific non programmable calculator.

Seat No:

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Student's Signature

Q.1 Answer the following: (Any Four)

8

1. State the model for growth of Mathematics.
2. If $\frac{d^2x}{dt^2} = a - 2bx$ then find the condition at which $\frac{d^2x}{dt^2} < 0$ and $\frac{d^2x}{dt^2} = 0$.
3. Let $x(t) = x(0)e^{3t}$. If $x(t) = 4$ then find $x(0)$.
4. State the equation of Orthogonal trajectories of the family of Concentric circles.
5. Find the Orthogonal trajectories of the family of curves $y = 2x$.
6. Write the differential equation of the family of Orthogonal trajectories of $\varphi\left(x, y, \frac{dy}{dx}\right) = 0$.

Q.2 Answer the following: (Any Two)

14

1. Derive the model for growth of Science and Scientists and solve it. Find the time t at which the number of scientists will become double of its present size.

2. Find the relation between doubling, tripling and quadrupling times for a population.
3. The rate of change of Atmospheric pressure P with respect to height h is assumed proportional to P . If $P = 14.7$ psi at $h = 0$ and $P = 7.35$ at $h = 17,500$ feet then find $P(10,000)$.

Q.3 Answer the following: (Any Two)

14

1. Derive the model of spread of Infectious diseases and solve it. Show that after a very long time all persons will be effected.
2. By using Logistic law of population growth model $\frac{x}{a-bx} = \left(\frac{x_0}{a-x_0}\right) e^{at}$, prove the $\lim_{t \rightarrow \infty} x(t) = \frac{a}{b}$.
3. Find the solution of a generalization of the logistic model $\frac{1}{N} \frac{dN}{dt} = \frac{r}{\alpha} \left(1 - \left(\frac{N}{K}\right)^\alpha\right), \alpha > 0$

Q.4 Answer the following: (Any Two)

14

1. Find the Orthogonal trajectories of the family of $x^2 + y^2 - 2ax = 0$.
2. Obtain the Orthogonal trajectories of the family of curves $r = ae^{\theta \cot \alpha}$ and $y^2 = 4cx$.
3. Find the Orthogonal trajectories of the family of Confocal conics $\frac{x^2}{a^2+\beta} + \frac{y^2}{b^2+\beta} = 1$, where β is a parameter.
